Motivation Some observations

Randomness and Universality inTopological Spaces

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Outline



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- Graph Theory
- Paper: Random Metric Spaces and Universality

2 Some observations

- On the Urysohn space
- Constructing Random Topological Spaces

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Graph Theory Paper: Random Metric Spaces and Universality

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The Erdös-Renyii Theorem

 The random graph on countably many vertices is universal with probability one.
 [P. Erdös, A. Rényi. Asymmetric graphs. Acta Math. Acad. Sci. Hungar. 14 (1963)]

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Main result of the paper:

 The random Polish Space is almost surely the Universal Urysohn Space.
 [A.M. Vershik, 2004]

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On the Urysohn space

• We consider Polish spaces as objects of a different category than metric spaces and isometries, for example, the category of uniform spaces and uniform isomorphisms or the category of topological spaces and homeomorphisms.

On the Urysohn space

- Every bi-Lipschitz function between finite subsets of the Urysohn space can be extended to a bi-Lipschitz function from the whole space onto itself. (follows from a theorem of W.Kubis, M.Rubin in *Extension* and reconstruction theorems for the Urysohn universal metric space, 2008)
- If we consider the sub-category of Polish spaces and uniform isomorphisms, the Universal Urysohn space satisfies universality and ω-homogeneity conditions.
- Similarly in the sub-category of Polish spaces and homeomorphisms, the Universal Urysohn space satisfies universality and ω-homogeneity conditions.

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Other ways to generate random spaces

• Initial Topologies:

- The random space generated this way is almost surely homeomorphic to the rationals.
- Every second countable,regular,countable space is homeomorphic to a subspace of the rationals Q.
 And Q is the unique (upto homeomorphism) space with this property that is strongly ω-homogeneous.
- *Random Closure Operators*: The random space almost always has the indiscrete topology.

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Rachel Alisha Zachariah Randomness and Universality